## Задача 1

Для задачи Штурма — Лиувилля с оператором $L$ на отрезке $[a,\, b]$, заданными граничными условиями и весом $\rho(x)$ найти собственные числа и собственные функции. (*2 балла*)

## Задача 2

Заданную функцию разложить в ряд по собственным функциям задачи Штурма — Лиувилля (см. предыдущую задачу). (*2 балла*)

| **Вар.** | $L$ | **Гр. условия** | $[a,b]$ | $\rho(x)$ | $f(x)$ |
|---|---|---|---|---|---|
| **1** | $-d^2/dx^2+I$ | $u(a)=u(b)=0$ | $[0,2]$ | 4 | $x^2-1$ |
| **2** | $-d^2/dx^2+2I$ | $u'(a)=u(b)=0$ | $[0,3]$ | 2 | $1-x^2$ |
| **3** | $-d^2/dx^2+3I$ | $u(a)=u'(b)=0$ | $[0,1]$ | 5 | $x^2+x$ |
| **4** | $-d^2/dx^2+4I$ | $u'(a)=u'(b)=0$ | $[0,4]$ | 3 | $x^2-x$ |
| **5** | $-d^2/dx^2+I$ | $u(a)=u(b)=0$ | $[0,\pi]$ | 4 | $\sin(x/2)$ |
| **6** | $-d^2/dx^2+2I$ | $u'(a)=u(b)=0$ | $[0,\pi]$ | 2 | $\cos(x/2)$ |
| **7** | $-d^2/dx^2+3I$ | $u(a)=u'(b)=0$ | $[0,\pi]$ | 5 | $\sin(x/2)$ |
| **8** | $-d^2/dx^2+4I$ | $u'(a)=u'(b)=0$ | $[0,\pi]$ | 3 | $\cos(x/2)$ |
| **9** | $-d^2/dx^2+I$ | $u(a)=u(b)=0$ | $[0,1]$ | 5 | $2x^2+1$ |
| **10** | $-d^2/dx^2+2I$ | $u'(a)=u(b)=0$ | $[0,2]$ | 4 | $1-2x^2$ |
| **11** | $-d^2/dx^2+3I$ | $u(a)=u'(b)=0$ | $[0,3]$ | 3 | $x^2+2x$ |
| **12** | $-d^2/dx^2+4I$ | $u'(a)=u'(b)=0$ | $[0,4]$ | 2 | $x^2-2x$ |
| **13** | $-d^2/dx^2+I$ | $u(a)=u(b)=0$ | $[0,\pi]$ | 7 | $\sin(x/2)-1$ |
| **14** | $-d^2/dx^2+2I$ | $u'(a)=u(b)=0$ | $[0,\pi]$ | 8 | $\cos(x/2)-1$ |
| **15** | $-d^2/dx^2+3I$ | $u(a)=u'(b)=0$ | $[0,\pi]$ | 9 | $\sin(x/2)+1$ |
| **16** | $-d^2/dx^2+4I$ | $u'(a)=u'(b)=0$ | $[0,\pi]$ | $\pi$ | $\cos(x/2)+1$ |
| **17** | $-d^2/dx^2+I$ | $u(a)=u(b)=0$ | $[0,4]$ | 4 | $2x^2+3$ |
| **18** | $-d^2/dx^2+2I$ | $u'(a)=u(b)=0$ | $[0,3]$ | 2 | $2-3x^2$ |
| **19** | $-d^2/dx^2+3I$ | $u(a)=u'(b)=0$ | $[0,2]$ | 5 | $3x^2+x$ |
| **20** | $-d^2/dx^2+4I$ | $u'(a)=u'(b)=0$ | $[0,1]$ | 3 | $3x^2-x$ |
| **21** | $-d^2/dx^2+I$ | $u(a)=u(b)=0$ | $[0,\pi/2]$ | 4 | $\sin(x/2)$ |
| **22** | $-d^2/dx^2+2I$ | $u'(a)=u(b)=0$ | $[0,\pi/2]$ | 2 | $\cos(x/2)$ |
| **23** | $-d^2/dx^2+3I$ | $u(a)=u'(b)=0$ | $[0,\pi/2]$ | 5 | $\sin(x/2)$ |
| **24** | $-d^2/dx^2+4I$ | $u'(a)=u'(b)=0$ | $[0,\pi/2]$ | 3 | $\cos(x/2)$ |
| **25** | $-d^2/dx^2+5I$ | $u(a)=u(b)=0$ | $[0,1]$ | 5 | $2x^2+1$ |
| **26** | $-d^2/dx^2+6I$ | $u'(a)=u(b)=0$ | $[0,2]$ | 4 | $1-2x^2$ |
| **27** | $-d^2/dx^2+7I$ | $u(a)=u'(b)=0$ | $[0,3]$ | 3 | $x^2+2x$ |
| **28** | $-d^2/dx^2+8I$ | $u'(a)=u'(b)=0$ | $[0,4]$ | 2 | $x^2-2x$ |
| **29** | $-d^2/dx^2+I$ | $u(a)=u(b)=0$ | $[0,\pi]$ | 4 | $\sin(x/2)+\pi$ |
| **30** | $-d^2/dx^2+2I$ | $u'(a)=u(b)=0$ | $[0,\pi]$ | 2 | $\cos(x/2)-\pi$ |

## Задача 3

Решить краевую задачу для уравнения Лапласа в прямоугольнике. (*2 балла*)

**1.** $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u\big|_{x=0} = \dfrac{y-b}{8b}, \ u'_x\big|_{x=a} = 0; \\ u'_y\big|_{y=0} = -\dfrac{1}{2a}\sin\dfrac{\pi x}{2a}, \ u\big|_{y=b} = \sin\dfrac{5\pi x}{2a}. \end{cases}$$

**2.** $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u'_x\big|_{x=0} = \dfrac{y(2b-y)}{4b^3}, \ u\big|_{x=a} = 0; \\ u'_y\big|_{y=0} = -\dfrac{3}{2a}\cos\dfrac{3\pi x}{2a}, \ u'_y\big|_{y=b} = \dfrac{1}{2a}\cos\dfrac{3\pi x}{2a}. \end{cases}$$

**3.** $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u\big|_{x=0} = \dfrac{y-b}{8b}, \ u\big|_{x=a} = 0; \\ u'_y\big|_{y=0} = -\dfrac{2}{a}\sin\dfrac{2\pi x}{a}, \ u\big|_{y=b} = \sin\dfrac{\pi x}{a}. \end{cases}$$

**4.** $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u'_x\big|_{x=0} = \dfrac{y-b}{2b}, \ u'_x\big|_{x=a} = 0; \\ u\big|_{y=0} = 1, \ u\big|_{y=b} = \cos\dfrac{\pi x}{a}. \end{cases}$$

**5.** $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u\big|_{x=0} = \dfrac{y-b}{2b}, \ u'_x\big|_{x=a} = 0; \\ u\big|_{y=0} = -\sin\dfrac{5\pi x}{2a}, \ u\big|_{y=b} = \sin\dfrac{\pi x}{2a}. \end{cases}$$

**6.** $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u'_x\big|_{x=0} = \dfrac{y(y-2b)}{64b^2}, \ u'_x\big|_{x=a} = 0; \\ u\big|_{y=0} = -\cos\dfrac{\pi x}{a}, \ u'_y\big|_{y=b} = \dfrac{3}{a}\cos\dfrac{3\pi x}{a}. \end{cases}$$

**7.** $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u\big|_{x=0} = \dfrac{y(y-2b)}{32b^2}, \ u\big|_{x=a} = 0; \\ u\big|_{y=0} = -\sin\dfrac{2\pi x}{a}, \ u'_y\big|_{y=b} = \dfrac{1}{a}\sin\dfrac{\pi x}{a}. \end{cases}$$

**8.** $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u\big|_{x=0} = -\cos\dfrac{5\pi y}{2b}, \ u'_x\big|_{x=a} = \dfrac{1}{2b}\cos\dfrac{\pi y}{2b}; \\ u'_y\big|_{y=0} = \dfrac{x(x-2a)}{64a^3}, \ u\big|_{y=b} = 0. \end{cases}$$

**9.** $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u'_x\big|_{x=0} = -\dfrac{1}{b}\cos\dfrac{\pi y}{b}, \ u\big|_{x=a} = 1; \\ u'_y\big|_{y=0} = \dfrac{x-a}{16a^2}, \ u'_y\big|_{y=b} = 0. \end{cases}$$

**10.** $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u\big|_{x=0} = -\cos\dfrac{\pi y}{2b}, \ u\big|_{x=a} = \cos\dfrac{\pi y}{2b}; \\ u'_y\big|_{y=0} = \dfrac{x-a}{2a^2}, \ u\big|_{y=b} = 0. \end{cases}$$

**11.** $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u'_x\big|_{x=a} = -\dfrac{1}{b}\sin\dfrac{\pi y}{b}, \ u'_x\big|_{x=a} = \dfrac{1}{3b}\sin\dfrac{\pi y}{b}; \\ u\big|_{y=0} = \dfrac{x(2a-x)}{4a^2}, \ u\big|_{y=b} = 0. \end{cases}$$

**12.** $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u\big|_{x=0} = -\sin\dfrac{\pi y}{b}, \ u'_x\big|_{x=a} = \dfrac{2}{b}\sin\dfrac{2\pi y}{b}; \\ u\big|_{y=0} = \dfrac{x(x-2a)}{32a^2}, \ u\big|_{y=b} = 0. \end{cases}$$

**13.** $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u'_x\big|_{x=0} = -\dfrac{1}{2b}\sin\dfrac{\pi y}{2b}, \ u'_x\big|_{x=a} = \dfrac{3}{2b}\sin\dfrac{3\pi y}{2b}; \\ u\big|_{y=0} = \dfrac{x(2a-x)}{4a^2}, \ u'_y\big|_{y=b} = 0. \end{cases}$$

**14.** $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u\big|_{x=0} = -\sin\dfrac{\pi y}{2b}, \ u\big|_{x=a} = \sin\dfrac{5\pi y}{2b}; \\ u\big|_{y=0} = \dfrac{x-a}{2a}, \ u'_y\big|_{y=b} = 0. \end{cases}$$

**15.** $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u\big|_{x=0} = 0, \ u'_x\big|_{x=a} = \dfrac{b-y}{16y_0^2}; \\ u'_y\big|_{y=0} = -\dfrac{3}{2a}\sin\dfrac{3\pi x}{2a}, \ u\big|_{y=b} = \sin\dfrac{\pi x}{2a}. \end{cases}$$

**16.** $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u'_x\big|_{x=0} = 0, \ u\big|_{x=a} = \dfrac{y(y-2b)}{4y_0^2}; \\ u'_y\big|_{y=0} = -\dfrac{1}{2a}\cos\dfrac{\pi x}{2a}, \ u'_y\big|_{y=b} = \dfrac{3}{2a}\cos\dfrac{3\pi x}{2a}. \end{cases}$$

**17.** $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u\big|_{x=0} = 0, \ \ u\big|_{x=a} = \dfrac{b-y}{8b}; \\ u'_y\big|_{y=0} = -\dfrac{4}{a}\sin\dfrac{4\pi x}{a}, \ \ u\big|_{y=b} = \sin\dfrac{\pi x}{a}. \end{cases}$$

**18.** $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u'_x\big|_{x=0} = 0, \ \ u'_x\big|_{x=a} = \dfrac{b-y}{2b^2}; \\ u\big|_{y=0} = -\cos\dfrac{2\pi x}{a}, \ \ u\big|_{y=b} = 1. \end{cases}$$

**19.** $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u\big|_{x=0} = 0, \ \ u'_x\big|_{x=a} = \dfrac{b-y}{2b^2}; \\ u\big|_{y=0} = \sin\dfrac{\pi x}{2a}, \ \ u\big|_{y=b} = \sin\dfrac{3\pi x}{2a}. \end{cases}$$

**20.** $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u'_x\big|_{x=0} = 0, \ \ u'_x\big|_{x=a} = \dfrac{y(2b-y)}{64y_0^3}; \\ u\big|_{y=0} = 1, \ \ u'_y\big|_{y=b} = \dfrac{1}{a}\cos\dfrac{\pi x}{a}. \end{cases}$$

**21.** $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u\big|_{x=0} = 0, \ \ u\big|_{x=a} = \dfrac{y(2b-y)}{32b^2}; \\ u\big|_{y=0} = -\sin\dfrac{2\pi x}{a}, \ \ u'_y\big|_{y=b} = \dfrac{2}{a}\sin\dfrac{2\pi x}{a}. \end{cases}$$

**22.** $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u\big|_{x=0} = -\cos\dfrac{\pi y}{2b}, \ \ u'_x\big|_{x=a} = \dfrac{1}{2b}\cos\dfrac{\pi y}{2b}; \\ u'_y\big|_{y=0} = 0, \ \ u\big|_{y=b} = \dfrac{x(2a-x)}{32a^2}. \end{cases}$$

**23.** $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u'_x\big|_{x=0} = -\dfrac{2}{b}\cos\dfrac{2\pi y}{b}, \ \ u\big|_{x=a} = 1; \\ u'_y\big|_{y=0} = 0, \ \ u'_y\big|_{y=b} = \dfrac{a-x}{16a^2}. \end{cases}$$

**24.** $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u\big|_{x=0} = -\cos\dfrac{3\pi y}{2b}, \ \ u\big|_{x=a} = \cos\dfrac{\pi y}{2b}; \\ u'_y\big|_{y=0} = 0, \ \ u\big|_{y=b} = \dfrac{a-x}{2a}. \end{cases}$$

**25.** $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u'_x\big|_{x=0} = -\dfrac{3}{b}\sin\dfrac{3\pi y}{b}, \ \ u'_x\big|_{x=a} = \dfrac{1}{b}\sin\dfrac{\pi y}{b}; \\ u\big|_{y=0} = 0, \ \ u\big|_{y=b} = \dfrac{x(x-2a)}{4a^2}. \end{cases}$$

**26.** $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u'_x\big|_{x=0} = \dfrac{y^2}{b^2}, \ \ u\big|_{x=a} = 0; \\ u'_y\big|_{y=0} = \dfrac{3}{2a}\cos\dfrac{3\pi x}{2a}, \ \ u'_y\big|_{y=b} = \dfrac{1}{2a}\cos\dfrac{\pi x}{2a}. \end{cases}$$

**27.** $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u'_x\big|_{x=0} = \dfrac{b^2-y^2}{8b^2}, \ \ u\big|_{x=a} = 0; \\ u'_y\big|_{y=0} = \dfrac{2}{a}\sin\dfrac{\pi x}{a}, \ \ u\big|_{y=b} = \sin\dfrac{2\pi x}{a}. \end{cases}$$

**28.** $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u'_x\big|_{x=0} = \dfrac{y(b-y)}{2b^2}, \ \ u'_x\big|_{x=a} = 0; \\ u\big|_{y=0} = 1, \ \ u\big|_{y=b} = \cos\dfrac{\pi x}{a}. \end{cases}$$

**29.** $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u'_x\big|_{x=0} = 1, \ \ u\big|_{x=a} = \dfrac{1}{b}\cos\dfrac{\pi y}{b}; \\ u'_y\big|_{y=0} = \dfrac{x^2-a^2}{16a^2}, \ \ u'_y\big|_{y=b} = 0. \end{cases}$$

**30.** $$\begin{cases} \Delta u = 0, \quad 0 < x < a, \quad 0 < y < b; \\ u'_x\big|_{x=0} = \dfrac{y(y-b)}{2b^2}, \ \ u'_x\big|_{x=a} = 0; \\ u\big|_{y=0} = \cos\dfrac{2\pi x}{a}, \ \ u\big|_{y=b} = \cos\dfrac{\pi x}{a}. \end{cases}$$

## Задача 4

Решить краевую задачу для уравнения Лапласа в круге. (*2 балла*)

**1.** $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 2, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=2} = 2\cos^3\varphi - \sin^3\varphi + \sin\varphi. \end{cases}$

**2.** $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 1, \quad 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=1} = 3\cos^3\varphi + \sin^3\varphi - \sin\varphi. \end{cases}$

**3.** $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 4, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=4} = \cos^3\varphi + 4\sin^3\varphi + \sin^2\varphi. \end{cases}$

**4.** $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 3, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=3} = 4\sin^3\varphi - \sin^2\varphi + \sin\varphi. \end{cases}$

**5.** $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 2, \quad 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=2} = 2\cos^3\varphi + \sin\varphi. \end{cases}$

**6.** $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 1, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=1} = \cos^3\varphi - \cos^2\varphi + \sin\varphi. \end{cases}$

**7.** $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 3, \quad 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=3} = 3\sin^3\varphi - \cos^3\varphi + \sin\varphi. \end{cases}$

**8.** $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 4, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=4} = 2\cos^3\varphi + 4\sin^3\varphi - \sin^2\varphi. \end{cases}$

**9.** $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 1, \quad 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=1} = -4\cos^3\varphi - \sin^3\varphi + 2\sin\varphi. \end{cases}$

**10.** $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 2, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=2} = 4\cos^3\varphi - 2\sin^3\varphi - 3\cos\varphi + 2\sin\varphi. \end{cases}$

**11.** $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 3, \quad 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=3} = \sin^3\varphi - \cos^3\varphi + 3\sin\varphi. \end{cases}$

**12.** $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 6, \quad 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=6} = 2\cos^3\varphi - 3\cos^3\varphi + \cos\varphi. \end{cases}$

**13.** $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 4, \quad 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=4} = 2\cos^3\varphi + 2\sin^3\varphi + 2\sin\varphi. \end{cases}$

**14.** $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 1, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=1} = -\sin^3\varphi - 2\sin^2\varphi + 3\sin\varphi. \end{cases}$

**15.** $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 2, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=2} = 4\cos^3\varphi - 2\sin^3\varphi + \cos^2\varphi. \end{cases}$

**16.** $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 3, \quad 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=3} = 2\cos^3\varphi - 4\sin^3\varphi + \cos\varphi. \end{cases}$

**17.** $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 4, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=4} = 2\cos^2\varphi - \sin^3\varphi + \cos\varphi. \end{cases}$

**18.** $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 5, \quad 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=5} = 2\sin^3\varphi - \sin\varphi + 2\cos\varphi. \end{cases}$

**19.** $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 2, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=2} = 2\cos^3\varphi + 4\sin^3\varphi + 2\sin\varphi. \end{cases}$

**20.** $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 1, \quad 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=1} = 6\sin^3\varphi - \cos^3\varphi + 4\sin\varphi. \end{cases}$

**21.** $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 3, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=3} = \cos^3\varphi + \sin^3\varphi + \sin^2\varphi. \end{cases}$

**22.** $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 4, \quad 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=4} = 4\sin^3\varphi - \sin\varphi - \cos\varphi. \end{cases}$

**23.** $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 5, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=5} = -\cos^3\varphi - \sin^3\varphi + \cos\varphi. \end{cases}$

**24.** $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 2, \quad 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=2} = 4\cos^3\varphi + 2\sin^3\varphi + \sin\varphi - 3\cos\varphi. \end{cases}$

**25.** $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 1, \quad 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=1} = 2\cos^3\varphi - \sin^3\varphi + 3\sin\varphi. \end{cases}$

**26.** $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 1, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=1} = \sin^3\varphi + 2\sin^2\varphi - 3\sin\varphi - \cos\varphi. \end{cases}$

**27.** $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 2, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=2} = \cos^3\varphi + \sin^3\varphi + 3\sin^2\varphi. \end{cases}$

**28.** $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 5, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=5} = 4\sin^3\varphi + \sin^2\varphi - \sin\varphi. \end{cases}$

**29.** $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 3, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=3} = 4\cos^3\varphi + 2\sin^3\varphi + 2\cos\varphi. \end{cases}$

**30.** $\begin{cases} \Delta u = 0, \quad 0 \leqslant r < 1, \quad 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=1} = 4\cos^3\varphi - 4\sin^3\varphi + \cos^2\varphi. \end{cases}$

## Задача 5

Решить краевую задачу для уравнения Гельмгольца в круге. (*2 балла*)

**1.** $\begin{cases} \Delta u + u = 0, & 0 \leqslant r < 2, \ 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=2} = 2\cos^3\varphi - 3\sin\varphi. \end{cases}$

**2.** $\begin{cases} \Delta u + u = 0, & 0 \leqslant r < 1, \ 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=1} = \cos^3\varphi + \sin\varphi. \end{cases}$

**3.** $\begin{cases} \Delta u + u = 0, & 0 \leqslant r < 2, \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=2} = 3\cos^3\varphi - \sin\varphi. \end{cases}$

**4.** $\begin{cases} \Delta u + u = 0, & 0 \leqslant r < 3, \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=3} = \cos^2\varphi - 3\sin\varphi. \end{cases}$

**5.** $\begin{cases} \Delta u + u = 0, & 0 \leqslant r < 2, \ 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=2} = \cos^3\varphi - 2\sin^3\varphi. \end{cases}$

**6.** $\begin{cases} \Delta u + u = 0, & 0 \leqslant r < 2, \ 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=2} = \sin^3\varphi + 5\cos\varphi. \end{cases}$

**7.** $\begin{cases} \Delta u + u = 0, & 0 \leqslant r < 1, \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=1} = \cos^2\varphi - 3\sin\varphi. \end{cases}$

**8.** $\begin{cases} \Delta u + u = 0, & 0 \leqslant r < 2, \ 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=2} = \cos^3\varphi + \sin\varphi. \end{cases}$

**9.** $\begin{cases} \Delta u + u = 0, & 0 \leqslant r < 3, \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=3} = \cos^3\varphi + \sin^2\varphi - \cos\varphi. \end{cases}$

**10.** $\begin{cases} \Delta u + u = 0, & 0 \leqslant r < 1, \ 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=1} = \sin^3\varphi + 3\cos\varphi. \end{cases}$

**11.** $\begin{cases} \Delta u + u = 0, & 0 \leqslant r < 1, \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=1} = \cos^2\varphi - 2\sin\varphi. \end{cases}$

**12.** $\begin{cases} \Delta u + u = 0, & 0 \leqslant r < 2, \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=2} = \cos^2\varphi + 5\sin\varphi. \end{cases}$

**13.** $\begin{cases} \Delta u + u = 0, & 0 \leqslant r < 2, \ 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=2} = \cos^3\varphi - \sin\varphi. \end{cases}$

**14.** $\begin{cases} \Delta u + u = 0, & 0 \leqslant r < 1, \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=1} = \cos^2\varphi - 6\sin^3\varphi. \end{cases}$

**15.** $\begin{cases} \Delta u + u = 0, & 0 \leqslant r < 2, \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=2} = \cos^3\varphi - \sin^2\varphi. \end{cases}$

**16.** $\begin{cases} \Delta u + u = 0, & 0 \leqslant r < 3, \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=3} = \cos^2\varphi - 3\sin\varphi. \end{cases}$

**17.** $\begin{cases} \Delta u + u = 0, & 0 \leqslant r < 2, \ 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=2} = 2\cos^3\varphi - \sin\varphi. \end{cases}$

**18.** $\begin{cases} \Delta u + u = 0, & 0 \leqslant r < 2, \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=2} = \cos^2\varphi + 5\sin\varphi. \end{cases}$

**19.** $\begin{cases} \Delta u + u = 0, & 0 \leqslant r < 2, \ 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=2} = 3\cos^3\varphi + 5\cos\varphi. \end{cases}$

**20.** $\begin{cases} \Delta u + u = 0, & 0 \leqslant r < 2, \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=2} = \cos^2\varphi + 4\sin\varphi. \end{cases}$

**21.** $\begin{cases} \Delta u + u = 0, & 0 \leqslant r < 1, \ 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=1} = \cos^3\varphi + \sin^3\varphi. \end{cases}$

**22.** $\begin{cases} \Delta u + u = 0, & 0 \leqslant r < 2, \ 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=2} = \cos^3\varphi + 12\sin^3\varphi. \end{cases}$

**23.** $\begin{cases} \Delta u + u = 0, & 0 \leqslant r < 3, \ 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=3} = \cos^3\varphi - \sin^3\varphi. \end{cases}$

**24.** $\begin{cases} \Delta u + u = 0, & 0 \leqslant r < 2, \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=2} = \cos^2\varphi - 3\sin^2\varphi. \end{cases}$

**25.** $\begin{cases} \Delta u + u = 0, & 0 \leqslant r < 2, \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=2} = \cos^2\varphi + 5\sin\varphi. \end{cases}$

**26.** $\begin{cases} \Delta u + u = 0, & 0 \leqslant r < 2, \ 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=2} = \cos^3\varphi - 2\sin\varphi. \end{cases}$

**27.** $\begin{cases} \Delta u + u = 0, & 0 \leqslant r < 1, \ 0 \leqslant \varphi < 2\pi; \\ u'_r\big|_{r=1} = \cos^2\varphi + 2\sin\varphi. \end{cases}$

**28.** $\begin{cases} \Delta u + u = 0, & 0 \leqslant r < 3, \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=3} = \sin^2\varphi - 3\sin\varphi. \end{cases}$

**29.** $\begin{cases} \Delta u + u = 0, & 0 \leqslant r < 2, \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=2} = \cos^3\varphi + 2\sin\varphi. \end{cases}$

**30.** $\begin{cases} \Delta u + u = 0, & 0 \leqslant r < 4, \ 0 \leqslant \varphi < 2\pi; \\ u\big|_{r=4} = \sin^2\varphi + 6\sin^3\varphi. \end{cases}$