**Задание № 4**

Решение задачи нелинейного программирования

Определить экстремум целевой функции вида

 = 1112+2222+1212+11+22

при условиях

111+122<=>1

211+222<=>2 .

1. Найти стационарную точку целевой функции и исследовать ее (функцию) на выпуклость (вогнутость) в окрестностях стационарной точки.
2. Составить функцию Лагранжа.
3. Получить систему неравенств в соответствии с теоремой Куна-Таккера.
4. Используя метод искусственных переменных составить симплекс-таблицу и найти решение полученной задачи линейного программирования.
5. Дать ответ с учетом условий дополняющей нежесткости.

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  № | *b*1 | *b*2 | *c*11 | *c*12 | *c*22 | extr | *a*11 | *a*12 | *a*21 | *a*22 | *p*1 | *p*2 | Знаки огр.  |
| 1 | 2 |
| 4 | –5 | –6 | 1 | –2 | 2 | min | 2 | 3 | 1 | 3 | 8 | 15 | ≤ | ≤ |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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**Решение: моя ПОПЫТКА ПЕРВАЯ**

 = 12+*2*22-*2*12-51-62 →min

 2x1+3x2≤8

 X1+3x2≤15

1. **Найдем стационарную точку**

F’x1=2x1-2x2-5=0 → 4x2-6-2x2-5=0

F’x2=4x2-2x1-6=0 → x1=2x2-3

 2x2=11 x2=11/2

 →

 X1=2x2-3 x1=8

M(8;11/2)- стационарная точка F

F”x1x1=2; F”x1x2=-2; F”x2x2=4

2 -2

 =8-4=4> 0

-2 4

Т.е. F вогнута и это задача квадратного программирования.

**2. Составим функцию Лагранжа.**

L=x12+2x22-2x1x2-5x1-6x2+ λ1(2x1+3x2-8)+ λ2(x1+3x2-15)

**3.Применим т. Куна-Такера, получим условия оптимального решения.**

L’x1=2x1-2x2-5+2 λ1+ λ2≤0

L’x2=2x2-2x1-6+3 λ1+3 λ2≤0

L’ λ1=2x1+3x2-8≥0 (1)

L’ λ2=x1+3x2-15≥0

И

L’x1\*x1= x1(2x1-2x2-5+2 λ1+ λ2)=0

 L’x2\*x2= x2(4x2-2x1-6+3λ1+3λ2)=0

 L’ λ1 \* λ1= λ1(2x1+3x2-8)=0

L’ λ2 \* λ2= λ2(x1+3x2-15)=0

4.

Вводим свободные переменные в систему. Перепишем (1) в каноническом виде:

2x1-2x2+2 λ1+λ2+V1=5

-2x1+4x2+3 λ1+3 λ2+ V2=6

2x1+3x2- ω 1=8

X1+3x2- ω2 =15

Решим симплекс методом.

Шаг№0

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| M | B | A0 | X1 | X2 | λ1 | λ2 | V1 | V2 | ω 1 | ω 2 |
| M | V1 | 5 | 2 | -2 | 2 | 1 | 1 | 0 | 0 | 0 |
| M | V2 | 6 | -2 | 4 | 3 | 3 | 0 | 1 | 0 | 0 |
| M | ω 1 | 8 | 2 | 3 | 0 | 0 | 0 | 0 | -1 | 0 |
| M | ω 2 | 15 | 1 | 3 | 0 | 0 | 0 | 0 | 0 | -1 |
|  |  | 34M | 3M | 8M | 5M | 4M | M | M | -M | -M |

Шаг№1

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| M | B | A0 | X1 | X2 | λ1 | λ2 | V1 | V2 | ω 1 | ω 2 |
| 0 | X1 | 5/2 | 1 | -1 | 1 | 1/2 | 1/2 | 0 | 0 | 0 |
| M | V2 | 11 | 0 | 2 | 5 | 4 | 1 | 1 | 0 | 0 |
| M | ω 1 | 3 | 0 | 5 | -2 | -1 | -1 | 0 | -1 | 0 |
| M | ω 2 | 12.5 | 0 | 4 | -1 | -1/2 | -1/2 | 0 | 0 | -1 |
|  |  | 26.5M | 0 | 11M | 2M | 5M/2 | -M/2 | M | -M | -M |

Шаг№2

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| M | B | A0 | X1 | X2 | λ1 | λ2 | V1 | V2 | ω 1 | ω 2 |
| 0 | x1 | 3.1 | 1 | 0 | 3/5 | 3/10 | 3/10 | 0 | 1/5 | 0 |
| M | V2 | 9.8 | 0 | 0 | 29/5 | 22/5 | 7/5 | 1 | 2/5 | 0 |
| 0 | X 2 | 0.6 | 0 | 1 | -2/5 | -1/5 | -1/5 | 0 | -1/5 | 0 |
| M | ω 2 | 10.1 | 0 | 0 | 3/5 | 3/10 | 3/10 | 0 | 4/5 | -1 |
|  |  | 19.9M | 0 | 0 | 32M/5 | 47M/10 | 17M/10 | M | 6M/5 | -M |

Шаг№3

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| M | B | A0 | X1 | X2 | λ1 | λ2 | V1 | V2 | ω 1 | ω 2 |
| 0 | x1 | 121/58 | 1 | 0 | 0 | -9/58 | 9/58 | -3/29 | 23/145 | 0 |
| 0 | λ1 | 49/29 | 0 | 0 | 1 | 22/29 | 7/29 | 5/29 | 2/29 | 0 |
| 0 | X 2 | 37/29 | 0 | 1 | 0 | 3/29 | 3/29 | 2/29 | -5/29 | 0 |
| M | ω 2 | 527/58 | 0 | 0 | 0 | -9/58 | 9/58 | -3/29 | 22/29 | -1 |
|  |  | 527M/58 | 0 | 0 | 0 | -9M/58 | 9M/58 | -3M/29 | 22M/29 | -M |

Шаг№4

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| M | B | A0 | X1 | X2 | λ1 | λ2 | V1 | V2 | ω 1 | ω 2 |
| 0 | x1 | -7 |  |  |  |  |  |  |  |  |
| 0 | λ1 | -112/9 |  |  |  |  |  |  |  |  |
| 0 | X 2 | -1249/261 |  |  |  |  |  |  |  |  |
| 0 | V 2 | 527/9 | 0 | 0 | 0 | -1 | 1 | -2/3 | 44/9 | -58/9 |
|  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Таблица оптимальна

X1= -7 λ1= -112/9

X2= -1249/261 λ2=0

Ответ:

fmin=49+2\*(1249/261)2 – 14\*(1249/261)+35+6\*(1249/261)=91.5

**Решение:моя ПОПЫТКА ВТОРАЯ**

 = 12+*2*22-*2*12-51-62 →min

 2x1+3x2≤8

 X1+3x2≤15

1. F’x1=2x1-2x2-5=0 → 4x2-6-2x2-5=0

F’x2=4x2-2x1-6=0 → x1=2x2-3

2x2=11 x2=11/2

 →

X1=2x2-3 x1=8

M(8;11/2)- стационарная точка F

F”x1x1=2; F”x1x2=-2; F”x2x2=4

2 -2

 =8-4=4> 0

2 4

Т.е. F вогнута и это задача квадратного программирования.

2. Составим функцию Лагранжа.

L(x1;x2;λ1; λ1)=x12+2x22-2x1x2-5x1-6x2+ λ1(2x1+3x2-8)+ λ2(x1+3x2-15)

3.Применим т. Куна-Такера, получим условия оптимального решения.

L’x1=2x1-2x2-5+2 λ1+ λ2≤0

L’x2=4x2-2x1-6+3 λ1+3 λ2≤0

L’ λ1=2x1+3x2-8=0

L’ λ2=x1+3x2-15=0

И условия дополняющей нежесткости

L’x1\*x1=0; L’x2\*x2=0; L’x1\* λ1=0

4.

Вводим свободные переменные в систему:

2x1-2x2-5+2 λ1+2 λ2+V1=0

-2x1+4x2-6+3 λ1+3 λ2+ V2=0

2x1+3x2-8=0

X1+3x2-15=0

Условия доп. Нежесткости

X1\* V1=0 x2\*V2=0

Эквивалентный вид:

2x1-2x2+2 λ1+2 λ2+V1=5 +y1

-2x1+4x2+3 λ1+3 λ2+ V2=6 +y2

2x1+3x2=8 +y3

X1+3x2=15 +y4

Решим симплекс методом.

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Cj/Ci |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | M | M | M | M |
|  | B | A0 | X1 | X2 | λ | λ | V1 | V2 | Y1 | Y2 | Y3 | Y4 |
| M | Y1 | 5 | 2 | -2 | 2 | 2 | 1 | 0 | 1 | 0 | 0 | 0 |
| M | Y2 | 6 | -2 | 4 | 3 | 3 | 0 | 1 | 0 | 1 | 0 | 0 |
| M | Y3 | 8 | 2 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| M | Y4 | 15 | 1 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
|  |  | 34M | 3M | 8M | 5M | 5M | M | M | 0 | 0 | 0 | 0 |

Таблица 2

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | B | A0 | X1 | X2 | λ | λ | V1 | V2 | Y1 | Y2 | Y3 | Y4 |
| 0 | X1 | 5/2 | 1 | -1 | 1 | 1 | 1/2 | 0 | 1/2 | 0 | 0 | 0 |
| M | Y2 | 11 | 0 | 2 | 5 | 5 | 1 | 1 | 1 | 1 | 0 | 0 |
| M | Y3 | 3 | 0 | 5 | -2 | -2 | -1 | 0 | -1 | 0 | 1 | 0 |
| M | Y4 | 25/2 | 0 | 4 | -1 | -1 | -1 | 0 | -1/2 | 0 | 0 | 0 |
|  |  | 26.5M | 0 | 11M | 2M | 2M | -M/2 | M | -3M/2 | 0 | 0 | 0 |

Таблица 3

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | B | A0 | X1 | X2 | λ | λ | V1 | V2 | Y1 | Y2 | Y3 | Y4 |
| 0 | X1 | 5 | 1 | 0 | 0 | 0 | 0.3 | -0.2 | 0.3 | -0.2 | 0 | 0 |
| 0 | λ | 11/5 | 0 | 0 | 1 | 0 | 1/5 | 1/5 | 1/5 | 1/5 | 0 | 0 |
| 0 | X2 | 2 | 0 | 1 | 0 | 0 | - | - | - | 0.4 | 1 | 0 |
| 0 | λ | 2/5 | 0 | 0 | 0 | 1 | - | - | - | -0.5 | - | - |
|  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

X1=5 λ1=11/5

X2=2 λ2=2/5

Ответ:

Fmin=25+8-20-25-12=-24