Ülesanne 1
Sümmeetrilise geomeetrilisel kujundi inertsimomendid

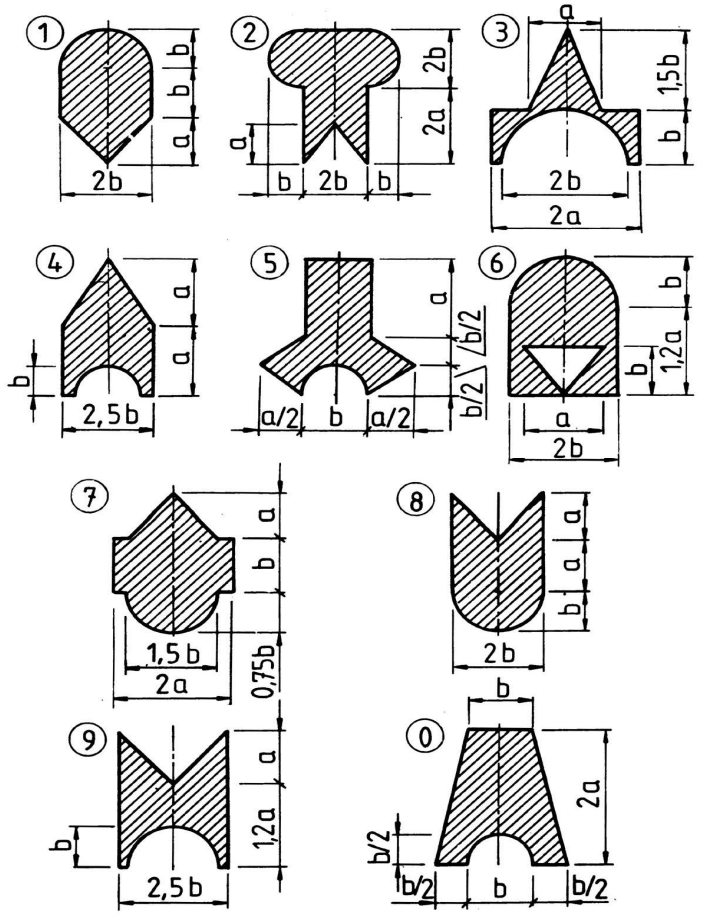
Arvutada joonisel 1 esitatud kujundi keskpeainertsimomendid.

Andmed

Kujundi skeem vastab indeksile **A.** **a** ja **b** võtta tabelist1.

Tabel 1

B	1	2	3	4	5	6	7	8	9	0
a, cm	8		9		10		11		12	
b, cm	6	7		8		9		10		11



Joonis 1

Nõutav lahenduskäik

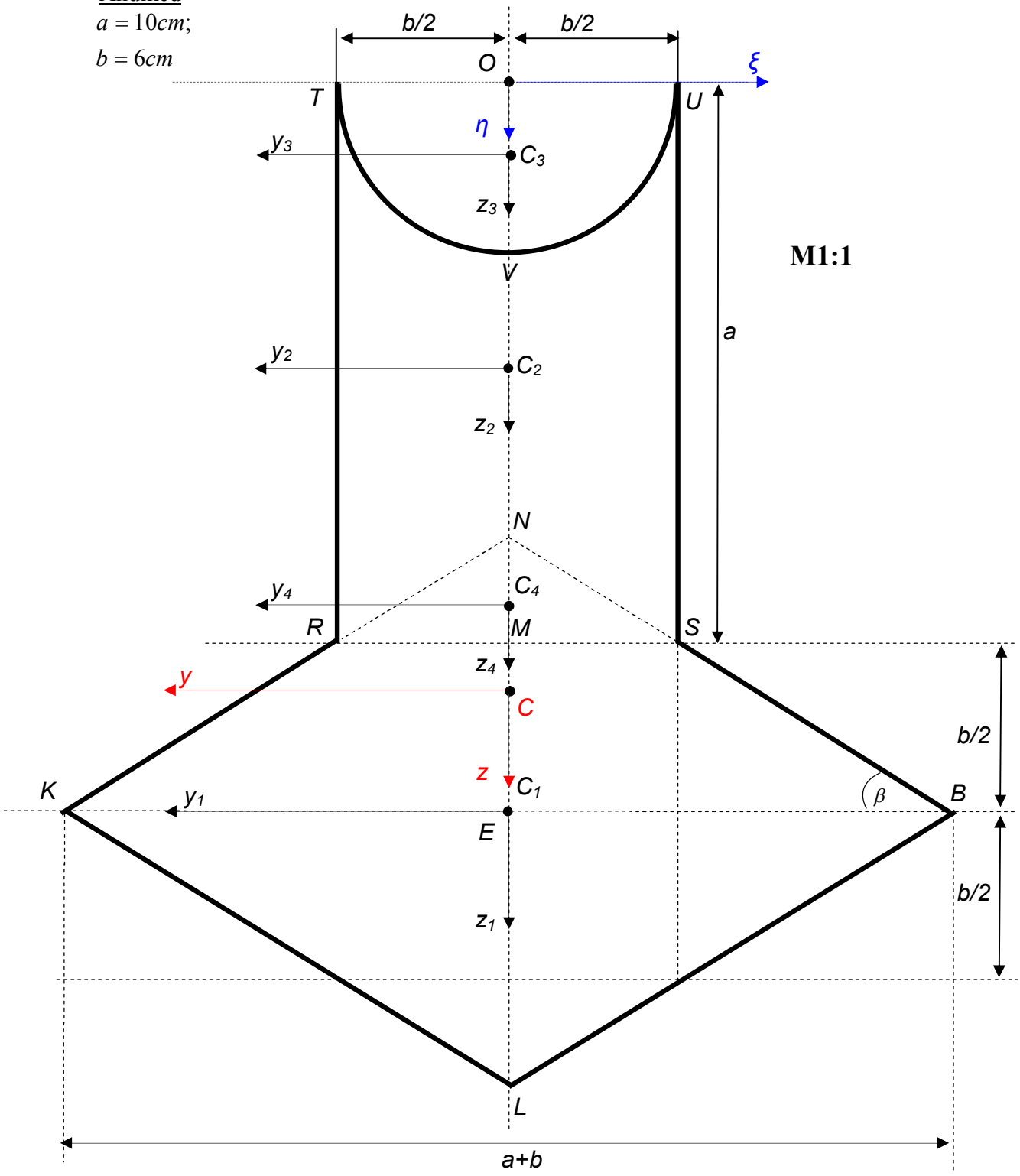
1. Määrata kujundi keskpeateljed.
2. Arvutada kujundi peainertsimomendid
3. Esitada sobivas mõõtkavas joonis, kus arvutustes kasutatud teljed

Lahendusnäide

Andmed

$a = 10cm;$

$b = 6cm$



Joonis 1a

Kuna η telg on kujundi sümmeetriatelg, siis ta on ka kujundi keskpeainertsitelg ja kujundi pinnakese peab asuma sellel teljel. Leiame kujundi pinnakeskme koordinaadi η_C kasutades negatiivsete pindalate meetodit.
Jooniselt on näha, et

$$BE = \frac{a+b}{2} = 8cm$$

$$\tan\beta = \frac{b}{a} = \frac{6}{10} = \frac{EL}{BE} = \frac{EL}{8}$$

Seega

$$EL = 4{,}8cm; \qquad LM = EL + EM = 4{,}8 + 3 = 7{,}8cm; \qquad MN = 1{,}8cm$$

Jaotame kujundi osakujunditeks järgmiselt
Osakujund nr. 1 – romb BLKN

$$\text{pindala } A_1 = \frac{KB \cdot LN}{2} = \frac{16 \cdot 9{,}6}{2} = 76{,}8cm^2$$

$$\eta_{C_1} = \eta_E = 4{,}8cm$$

Osakujund nr. 2 – ristkülik SRTU

$$\text{pindala } A_2 = ab = 60cm^2$$

$$\eta_{C_2} = LM + \frac{a}{2} = 7{,}8 + 5 = 12{,}8cm$$

Osakujund nr. 3 – poolring TOUV

$$\text{pindala } A_3 = -\frac{\pi \cdot b^2}{8} = -4{,}5\pi = -14{,}14cm^2$$

$$OC_3 = \frac{2b}{3\pi} = 1{,}273cm$$

$$\eta_{C_3} = LM + a - KC_3 = 16{,}53cm$$

Osakujund nr.4 – kolmnurk RSN

$$\text{pindala } A_3 = -\frac{b \cdot MN}{2} = -\frac{6 \cdot 1{,}8}{2} = -5{,}4cm^2$$

$$\eta_{C_4} = LM + \frac{1}{3}MN = 7{,}8 + 0{,}6 = 8{,}4cm$$

Seega

$$\eta_C = \frac{A_1\eta_{C_1} + A_2\eta_{C_2} + A_3\eta_{C_3} + A_4\eta_{C_4}}{A_1 + A_2 + A_3 + A_4} =$$

$$= \frac{76{,}8 \cdot 4{,}8 + 60 \cdot 12{,}8 - 14{,}14 \cdot 16{,}53 - 5{,}4 \cdot 8{,}4}{76{,}8 + 60 - 14{,}14 - 5{,}4} = \frac{857{,}55}{117{,}26} = 7{,}31cm$$

Tähistades sümboliga $I^{(n)}$ osakujundi nr. n inertsimomendi saame

$$I_z = I_z^{(1)} + I_z^{(2)} - I_z^{(3)} - I_z^{(4)}$$
$$I_y = I_{y_1}^{(1)} + I_{y_2}^{(2)} - I_{y_3}^{(3)} - I_{y_4}^{(4)} + A_1 z_{C_1}^2 + A_2 z_{C_2}^2 + A_3 z_{C_3}^2 + A_4 z_{C_4}^2$$

kus

$$z_{C_n} = \eta_C - \eta_{C_n}$$

Seega

$$z_{C_1} = 2{,}51cm$$
$$z_{C_2} = -5{,}49cm$$
$$z_{C_3} = -9{,}29cm$$
$$z_{C_3} = -1{,}09cm$$

Arvestades, et

$$I_z^{(1)} = 2\frac{LN \cdot (EB)^3}{12} = 819{,}2cm^4$$
$$I_z^{(2)} = \frac{ab^3}{12} = 180cm^4$$
$$I_z^{(3)} = \frac{\pi b^4}{128} = 31{,}81cm^4$$
$$I_z^{(4)} = 2\frac{MN \cdot (MS)^3}{48} = 2{,}025cm^4$$

Saame $I_z = 965{,}4\ \text{cm}^4$

Kuna

$$I_{y_1}^{(1)} = 2\frac{KB \cdot (EL)^3}{12} = 2\frac{16 \cdot 4{,}8^3}{12} = 294{,}91cm^4$$
$$I_{y_2}^{(2)} = \frac{ba^3}{12} = 500cm^4$$
$$I_{y_3}^{(3)} \approx 0{,}00686b^4 = 8{,}89cm^4$$
$$I_{y_4}^{(4)} = \frac{b \cdot (MN)^3}{36} = 0{,}972cm^4$$

Saame

$$I_y = 294{,}972 + 500 - 8{,}89 - 0{,}972 +$$
$$+ 76.8 \cdot 2{,}513^2 + 60 \cdot 5{,}487^2 - 14{,}14 \cdot 9{,}287^2 - 5{,}4 \cdot 1{,}087^2 = 1851cm^4$$

Seega

I_z= 965,4 cm^4**=9,654**E+02 cm^4

I_y= 1851 cm^4**=1,851**E+03 cm^4

Tabel 1a

B	A →	1	2	3	4	5	6	7	8	9	0
↓1	I_y, cm^4	3.800E+03	1.864E+04	8.182E+02	1.375E+03	1.317E+03	2.737E+03	2.370E+03	5.369E+03	1.989E+03	2.415E+03
	I_z, cm^4	1.661E+03	1.229E+04	1.635E+03	2.304E+03	6.922E+02	1.827E+03	2.331E+03	2.625E+03	3.879E+03	1.048E+03
2	I_y, cm^4	6.115E+03	2.739E+04	1.033E+03	1.522E+03	1.761E+03	3.892E+03	3.707E+03	7.180E+03	2.187E+03	2.747E+03
	I_z, cm^4	3.001E+03	2.223E+04	1.558E+03	3.523E+03	1.049E+03	3.063E+03	2.881E+03	4.144E+03	6.024E+03	1.656E+03
3	I_y, cm^4	6.799E+03	3.264E+04	1.401E+03	2.254E+03	2.256E+03	4.759E+03	4.211E+03	9.198E+03	3.257E+03	3.988E+03
	I_z, cm^4	3.058E+03	2.263E+04	2.619E+03	4.082E+03	1.221E+03	3.306E+03	3.917E+03	4.544E+03	6.895E+03	1.870E+03
4	I_y, cm^4	1.029E+04	4.568E+04	1.678E+03	2.464E+03	2.909E+03	6.466E+03	6.222E+03	1.186E+04	3.539E+03	4.458E+03
	I_z, cm^4	5.107E+03	3.784E+04	2.462E+03	5.892E+03	1.754E+03	5.173E+03	4.721E+03	6.984E+03	1.009E+04	2.779E+03
5	I_y, cm^4	1.130E+04	5.333E+04	2.239E+03	3.498E+03	3.625E+03	7.733E+03	6.961E+03	1.478E+04	5.050E+03	6.224E+03
	I_z, cm^4	5.192E+03	3.844E+04	3.975E+03	6.725E+03	2.007E+03	5.538E+03	6.202E+03	7.582E+03	1.139E+04	3.099E+03
6	I_y, cm^4	1.631E+04	7.189E+04	2.582E+03	3.789E+03	4.542E+03	1.014E+04	9.842E+03	1.852E+04	5.437E+03	6.865E+03
	I_z, cm^4	8.166E+03	6.051E+04	3.705E+03	9.289E+03	2.767E+03	8.221E+03	7.328E+03	1.108E+04	1.593E+04	4.395E+03
7	I_y, cm^4	1.773E+04	8.259E+04	3.395E+03	5.198E+03	5.536E+03	1.192E+04	1.088E+04	2.257E+04	7.496E+03	9.285E+03
	I_z, cm^4	8.287E+03	6.136E+04	5.784E+03	1.048E+04	3.123E+03	8.742E+03	9.365E+03	1.193E+04	1.778E+04	4.851E+03
8	I_y, cm^4	2.466E+04	1.080E+05	3.803E+03	5.587E+03	6.782E+03	1.521E+04	1.485E+04	2.765E+04	8.013E+03	1.014E+04
	I_z, cm^4	1.243E+04	9.210E+04	5.362E+03	1.398E+04	4.165E+03	1.245E+04	1.089E+04	1.676E+04	2.400E+04	6.630E+03
9	I_y, cm^4	2.658E+04	1.225E+05	4.936E+03	7.452E+03	8.119E+03	1.760E+04	1.626E+04	3.310E+04	1.074E+04	1.335E+04
	I_z, cm^4	1.259E+04	9.327E+04	8.133E+03	1.560E+04	4.648E+03	1.317E+04	1.361E+04	1.793E+04	2.654E+04	7.255E+03
0	I_y, cm^4	3.585E+04	1.563E+05	5.408E+03	7.959E+03	9.764E+03	2.196E+04	2.156E+04	3.981E+04	1.141E+04	1.445E+04
	I_z, cm^4	1.817E+04	1.347E+05	7.516E+03	2.025E+04	6.036E+03	1.813E+04	1.561E+04	2.438E+04	3.480E+04	9.623E+03